

THE TRANSPORTATION PROBLEM

BASIC CONCEPTS AND FORMULA

Basic Concepts

1. Transportation Problem:

This type of problem deals with optimization of transportation cost in a distribution scenario involving m factories (sources) to n warehouses (destination) where cost of shipping from i th factory to j th warehouse is given and goods produced at different factories and requirement at different warehouses are given.

2. Northwest corner Rule:

The idea is to find an initial basic feasible solution i.e., a set of allocations that satisfied the row and column totals. This method simply consists of making allocations to each row in turn, apportioning as much as possible to its first cell and proceeding in this manner to its following cells until the row total is exhausted.

3. Algorithm Involved Under North-West Corner Rule

Steps:

1. Before allocation ensure that the total on demand & supply of availability and requirement are equal. If not then make same equal.
2. The first allocation is made in the cell occupying the upper left hand corner of the matrix.

The assignment is made in such a way that either the resource availability is exhausted or the demand at the first destination is satisfied.

3. (a) If the resource availability of the row one is exhausted first, we move down the second row and first column to make another allocation which either exhausts the resource availability of row two or satisfies the remaining destination demand of column one.
(b) If the first allocation completely satisfies the destination demand of column one, we move to column two in row one, and make a second allocation which either exhausts the remaining resource availability

of row one or satisfies the destination requirement under column two.

4. The Least Cost Method:

- i) Before starting the process of allocation ensure that the total of availability and demand is equal. The least cost method starts by making the first allocation in the cell whose shipping cost (or transportation cost) per unit is lowest.
- ii) This lowest cost cell is loaded or filled as much as possible in view of the origin capacity of its row and the destination requirements of its column.
- iii) We move to the next lowest cost cell and make an allocation in view of the remaining capacity and requirement of its row and column. In case there is a tie for the lowest cost cell during any allocation, we can exercise our judgment and we arbitrarily choose cell for allocation.
- iv) The above procedure is repeated till all row requirements are satisfied.

5. Vogel's Approximation Method (VAM)

VAM entails the following steps:

Step 1: For each row of the transportation table identify the smallest and next smallest costs. Find the difference between the two costs and display it to the right of that row as "Difference" (Diff.). Likewise, find such a difference for each column and display it below that column. In case two cells contain the same least cost then the difference will be taken as zero.

Step 2: From amongst these row and column differences, select the one with the largest difference. Allocate the maximum possible to the least cost cell in the selected column or row. If there occurs a tie amongst the largest differences, the choice may be made for a row or column which has least cost. In case there is a tie in cost cell also, choice may be made for a row or column by which maximum requirement is exhausted. Match that column or row containing this cell whose totals have been exhausted so that this column or row is ignored in further consideration.

Step 3: Recompute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the column and row totals are exhausted.

6. Optimality Test

Once the initial allocation is done, we have to do the optimality test if it satisfy the condition that number of allocation is equal to $(m+n-1)$ where m = number of rows, n = number of columns. If allocation is less than $(m+n-1)$, then the problem shows

degenerate situation. In that case we have to allocate an infinitely small quantity (ϵ) in least cost and independent cell.

7. Cell Evaluations

The allocations are $m+n-1$ in number and independent.

For each allocated cell, cell value = $c_{ij} = u_i + v_j$ where u_i = row value + column value.

One row where maximum allocation is made, U value is made zero and u_i and v_j for all rows and columns are calculated.

For each unallocated cell, cell value = [cost of cell $-(u + v)$]

Question 1

A product is manufactured by four factories A, B, C and D. The Unit production costs are Rs.2, Rs.3, Rs.1 and Rs.5 respectively. Their daily production capacities are 50, 70, 30 and 50 units respectively. These factories supply the product to four P, Q, R and S. The demand made by these stores are 25, 35, 105 and 20 Units transportation cost in rupees from each factory to each store is given in the following table;

		Stores			
		P	Q	R	S
Factory	A	2	4	6	11
	B	10	8	7	5
	C	13	3	9	12
	D	4	6	8	3

Determine the extent of deliveries from each of the factories to each of the stores so that the total cost (production and transportation together) is minimum.

Answer

The new transportation costs table, which consists of both production and transportation costs, is given in following table.

		Store			
		P	Q	R	S
Factories	A	2+2=4	4+2=6	6+2=8	11+2=13
	B	10+3=13	8+3=11	7+3=10	5+3=8
	C	13+1=14	3+1=4	9+1=10	12+1=13
					Supply
					50
					70
					30

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D	4+5=9	6+5=11	8+5=13	3+5=8	50
Demand	25	35	105	20	200
					185

Since the total supply of 200 units exceeds the total demand of 185 units by $200-185=15$ units of product, there fore a dummy destination (store) is added to absorb the excess supply. The associated cost coefficients in dummy store are taken as zero as the surplus quantity remains lying in the respective factories and is, in fact, not shipped at all. The modified table is given below. The problem now becomes a balanced transportation one and it is a minimization problem. We shall now apply Vogel's Approximation method to find an initial solution.

	P	Q	R	S	Dummy	Supply	Difference
A	25	5	20	13	0	50/25/20/0	4 2 2 2 5
	4	6	8				
B	13	11	70	8	0	70/0	8 2 2 2 2 2
			10				
C	14	30	10	13	0	30/0	4 6 _ _ _ _
		4					
D	9	11	15	20	15	50/35/15/0	8 1 1 3 3 5
			13	8	0		
Demand	25/0	35/5/0	105/85/15/0	20/0	15/0	200	
Difference	5	2	2	0	0		
	5	2	2	0	-		
	5	5	2	0	-		
	-	5	2	0	-		
	-	-	2	0	-		

The initial solution is shown in above table. It can be seen that 15 units are allocated to dummy store from factory D. This means that the company may cut down the production by 15 units at the factory where it is uneconomical. We will now test the optimality of the solution. The total number of allocations is 8 which is equal to the required $m+n-1 (=8)$ allocation. Introduce u_i 's, v_j 's, $i = (1,2,- - - -4)$ and $j = (1,2,- - - -5)$ $\Delta_{ij}=C_{ij}-(u_i+v_j)$ for allocated cells. We assume that $u_4=0$ and remaining u_j 's, v_j 's and Δ_{ij} 's are calculated below."

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	P	Q	R	S	Dummy	Supply	U_i
A	25 4	5 6	20 8	13 +10	0 +5	50	$U_1 = -5$
B	13 +7	11 +3	70 10	8 +3	0 +3	70	$U_2 =$
C	14 +1	30 4	10 +4	13 +12	0 +7	30	$U_3 = -7$
D	9 0	11 0	15 13	20 8	15 0	50	$U_4 = 0$
Demand	25	35	105	20	15		
V_j	$V_1=9$	2	2	0	0		

Please note that figures in top left hand corners of the cell represent the cost and the one in the bottom right hand corner of the non basic cell are the values of $\Delta_{ij} = c_{ij} - [(u_i + v_j)]$.

Since opportunity cost in all the unoccupied cells is positive, therefore initial solution is an optimal solution also. The total cost (transportation and production together) associated with this solution is

$$\begin{aligned}
 \text{Total cost} &= 4 \times 25 + 6 \times 5 + 8 \times 20 + 10 \times 70 + 4 \times 30 + 13 \times 15 + 8 \times 20 + 0 \times 15 \\
 &= 100 + 30 + 160 + 700 + 120 + 195 + 160 \\
 &= \text{Rs.1,465/-}
 \end{aligned}$$

Question 2

A compressed Natural Gas (CNG) company has three plants producing gas and four outlets. The cost of transporting gas from different production plants to the outlets, production capacity of each plant and requirement at different outlets is shown in the following cost-matrix table:

Plants	Outlets				Capacity of Production
	A	B	C	D	
X	4	6	8	6	700
Y	3	5	2	5	400
Z	3	9	6	5	600
Requirement	400	450	350	500	1,700

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Determine a transportation schedule so that the cost is minimized.

The cost in the cost-matrix is given in thousand of rupees.

Answer

The given problem is a balanced minimization transportation problem. The objective of the company is to minimize the cost. Let us find the initial feasible solution using Vogel's Approximation method (VAM)

Plants	Outlets				Capacity	Difference
	A	B	C	D		
X	4	400 6	8	300 6	700/300/0	2 2 0 0
Y	3	50 5	350 2	5	400/50/0	1 2 0 0
Z	400 3	9	6	200	600/200/0	2 2 4 0
Requirement	400/0	450/400/0	350/0	500/300/0		
Difference	0	1	4	0		
	0	1	-	0		
	-	1	-	0		

The initial feasible solution obtained by VAM is given below:

Plants	Outlets				Capacity
	A	B	C	D	
X	4	400 6	8	300 6	700
Y	3	50 5	350 2	5	400
Z	400 3	9	6	200 5	600
Requirement	400	450	350	500	

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Since the number of allocations = 6 = (m+n-1), let us test the above solution for optimality. Introduce u_i ($i=1,2,3$) and v_j ($j=1,2,3,4$) such that $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for allocated cells. We assume $u_1=0$, and rest of the u_i 's, v_j 's and Δ_{ij} 's are calculated as below:

		Outlets				
Plants		A	B	C	D	U_i
X		0 4	400 6	5 8	300 6	0
Y		0 3	50 5	350 2	0 5	-1
Z		400 3	4 9	4 6	200 5	-1
V_j		4	6	3	6	

On calculating Δ_{ij} 's for non-allocated cells, we found that all the $\Delta_{ij} \geq 0$, hence the initial solution obtained above is optimal.

The optimal allocations are given below.

Plants	Outlet	Units		Cost		Total Cost
X	→B	400	×	6	=	2,400
X	→D	300	×	6	=	1,800
Y	→B	50	×	5	=	250
Y	→C	350	×	2	=	700
Z	→A	400	×	3	=	1,200
Z	→D	200	×	5	=	<u>1,000</u>
						<u>7,350</u>

The minimum cost = 7,350 thousand rupees.

Since some of the Δ_{ij} 's = 0, the above solution is not unique. Alternative solutions exist.

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Question 3

Consider the following data for the transportation problem:

Factory	Destination			Supply to be exhausted
	(1)	(2)	(3)	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Demand	75	20	50	

Since there is not enough supply, some of the demands at the three destinations may not be satisfied. For the unsatisfied demands, let the penalty costs be rupees 1, 2 and 3 for destinations (1), (2) and (3) respectively.

Answer

The initial solution is obtained below by vogel's method.

Since demand ($=75+20+50=145$) is greater than supply ($=10+80+15=105$) by 40 units, the given problem is an unbalanced one. We introduce a dummy factory with a supply of 40 units. It is given that for the unsatisfied demands, the penalty cost is rupees 1, 2, and 3 for destinations (1), (2) and (3) respectively. Hence, the transportation problem becomes

Factory	Destination			Supply to be exhausted
	(1)	(2)	(3)	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Dummy	1	2	3	40
Demand	75	20	50	145

		Destination			Supply	Difference
		(1)	(2)	(3)		
Factory	A	5	10	7	100	4 _ _
	B	20	10	50	80/70/50/0	2 2 2
		6	4	6		

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C	15				15/0	1 1 1
		3		2	5	
Dummy	40				40/0	1 1 _
		1		2	3	
Demand	75/35/20/0	20/10/0	50/0			
Difference	2	1	2			
	2	0	2			
	3	2	1			

The initial solution is given in the table below.

		Destination			Supply
		(1)	(2)	(3)	
Factory	A		10		10
		5	1	7	
	B	20	10	50	80
		6	4	6	
C		15			15
		3	2	5	
	Dummy	40			40
		1	2	3	
Demand		75	20	50	

We now apply the optimality test to find whether the initial solution found above is optimal or not.

The number of allocations is 6 which is equal to the required $m+n-1$ (=6) allocations. Also, these allocations are independent. Hence, both the conditions are satisfied.

Let us now introduce u_i , and v_j $i = (1,2,3,4)$ and $j = (1,2,3)$ such that $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for allocated cells. We assume that $u_2 = 0$ and remaining u_i 's, v_j 's and Δ_{ij} 's are calculated as below:-

		(1)	(2)	(3)	u_i 's
A		2	10	4	-3
		5	1	7	

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Factory	B	20		10		50		0
		6		4		6		
	C	15		1		2		-3
		3		2		5		
Dummy		40		3		2		-5
		1		2		3		
vj's		6		4		6		

Since all Δ_{ij} 's for non basic cells are positive, therefore, the solution obtained above is an optimal one. The allocation of factories to destinations and their cost is given below:-

Factory	Destination	Units	Cost	Total Cost	
A	(2)	10	Re 1	Rs.10	Transportation Cost
B	(1)	20	Rs.6	Rs.120	
B	(2)	10	Rs.4	Rs.40	
B	(3)	50	Rs.6	Rs.300	
C	(1)	15	Rs.3	Rs.45	Penalty Cost
Dummy	(1)	40	Re 1	Rs.40	
				Rs.555	

Question 4

A manufacturing company produces two types of product the SUPER and REGULAR. Resource requirements for production are given below in the table. There are 1,600 hours of assembly worker hours available per week. 700 hours of paint time and 300 hours of inspection time. Regular customers bill demand at least 150 units of the REGULAR type and 90 units of the SUPER type. (8 Marks)

Table

Product	Profit/contribution Rs.	Assembly time Hrs.	Paint time Hrs.	Inspection time Hrs.
REGULAR	50	1.2	0.8	0.2
SUPER	75	1.6	0.9	0.2

Formulate and solve the given Linear programming problem to determine product mix on a weekly basis.

Answer

Let x_1 and x_2 denote the number of units produced per week of the product 'REGULAR' and 'SUPER' respectively.

Maximise $Z = 50x_1 + 75x_2$

Subject to

$$1.2x_1 + 1.6x_2 \leq 1,600 \text{ or } 12x_1 + 16x_2 \leq 16,000 \quad \text{-(i)}$$

$$0.8x_1 + 0.9x_2 \leq 700 \text{ or } 8x_1 + 9x_2 \leq 7,000 \quad \text{-(ii)}$$

$$0.2x_1 + 0.2x_2 \leq 300 \quad \text{or } 2x_1 + 2x_2 \leq 3,000 \quad \text{-(iii)}$$

$$x_1 \geq 150 \quad \text{-(iv)}$$

$$x_2 \geq 90 \quad \text{-(v)}$$

Let

$$x_1 = y_1 + 150$$

$$x_2 = y_2 + 90 \text{ where } y_1, y_2 \geq 0$$

$$\text{Maximize } Z = 50(y_1 + 150) + 75(y_2 + 90) \text{ or } Z = 50y_1 + 75y_2 + 14,250$$

Subject to:

$$12(y_1 + 150) + 16(y_2 + 90) \leq 16,000$$

$$8(y_1 + 150) + 9(y_2 + 90) \leq 7,000$$

$$2(y_1 + 150) + 2(y_2 + 90) \leq 3,000$$

$$\text{and } y_1, y_2 \geq 0$$

Adding slack variables s_1, s_2, s_3 , we get

Maximize $Z = 50y_1 + 75y_2 + 14,250$ subject to

$$12y_1 + 16y_2 + s_1 = 12,760$$

$$8y_1 + 9y_2 + s_2 = 4,990$$

$$2y_1 + 2y_2 + s_3 = 2,520$$

Table -1

C_j			50	75	0	0	0	
C_b			y_1	y_2	s_1	s_2	s_3	
0	s_1	12,760	12	16	1	0	0	12760/16

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0	s_2	4,990	8	9	0	1	0	4990/9
0	s_3	2,520	2	2	0	0	1	2520/2
Δ_j			-50	-75	0	0	0	

Table II

C_j			50	75	0	0	0
C_b			y_1	y_2	s_1	s_2	s_3
0	s_1	3889	-20/9	0	1	-16/9	0
75	y_2	554.44	8/9	1	0	1/9	0
0	s_3	1411	2/9	0	0	-2/9	1
Δ_j			50/3	0	0	75/9	0

Since all the elements in the index row are either positive or equal to zero, table II gives an optimum solution which is $y_1 = 0$ and $y_2 = 554.44$

Substituting these values we get

$$x_1 = 0 + 150 = 150$$

$$x_2 = 90 + 554.44 = 644.44 \quad \text{and the value of objective function is}$$

$$Z = 50 \times 150 + 75 \times 644.44$$

$$= \text{Rs. } 55,833$$

Question 5

A company manufactures two products A and B, involving three departments – Machining, Fabrication and Assembly. The process time, profit/unit and total capacity of each department is given in the following table:

	Machining (Hours)	Fabrication (Hours)	Assembly (Hours)	Profit (Rs).
A	1	5	3	80
B	2	4	1	100
Capacity	720	1,800	900	

Set up Linear Programming Problem to maximise profit. What will be the product Mix at Maximum profit level ?

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Answer

Maximize $z = 80x + 100y$ subject to

$$\begin{aligned} x + 2y &\leq 720 \\ 5x + 4y &\leq 1800 \\ 3x + y &\leq 900 \\ x \geq 0, y &\geq 0 \end{aligned}$$

where

$$\begin{aligned} x &= \text{No. of units of A} \\ y &= \text{No. of units of B} \end{aligned}$$

By the addition of slack variables s_1, s_2 and s_3 the inequalities can be converted into equations. The problems thus become

$$\begin{aligned} z = 80x + 100y \text{ subject to } & x + 2y + s_1 = 720 \\ & 5x + 4y + s_2 = 1800 \\ & 3x + y + s_3 = 900 \\ \text{and } x \geq 0, y \geq 0, & s_1 \geq 0, s_2 \geq 0, s_3 \geq 0 \end{aligned}$$

Table I:

	Profit/unit	Qty.	80	100	0	0	0	
			X	Y	S_1	S_2	S_3	
S_1	0	720	I	2	1	0	0	$\frac{720}{2} = 360$
S_2	0	1800	5	4	0	1	0	$1800/4 = 450$
S_3	0	900	3	I	0	0	1	$900/1 = 900$
Net evaluation row			80	100	0	0	0	
$1800 - 720 \times 4/2 = 360$			$900 - 720 \times 1/2 = 540$					
$5 - 1 \times 2 = 3$			$3 - 1 \times 1/2 = 5/2$					
$4 - 2 \times 2 = 0$			$1 - 2 \times 1/2 = 0$					
$0 - 1 \times 2 = -2$			$0 - 1 \times 1/2 = -1/2$					
$1 - 0 \times 2 = 1$			$0 - 0 \times 1/2 = 0$					
$0 - 0 \times 2 = 0$			$1 - 0 \times 1/2 = 1$					

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Table 2:

			80	100	0	0	0	
Program	Profit/unit	Qty.	X	Y	S ₁	S ₂	S ₃	
Y	100	360	½	I	½	0	0	360 ÷ ½ = 720
S2	0	360	3	0	-2	1	0	360 ÷ 3 = 120
S3	0	540	5/2	0	-1/2	0	I	540 ÷ 5/2 = 216
Net evaluation row			30	0	-50	0	0	

$$360 - 360 \times 1/6 = 300$$

$$\frac{1}{2} - 3 \times 1/6 = 0$$

$$1 - 0 \times 1/6 = 1$$

$$\frac{1}{2} - 2 \times 1/6 = 5/6$$

$$0 - 1 \times 1/6 = -1/6$$

$$0 - 0 \times 1/6 = 0$$

$$540 - 360 \times 5/6 = 240$$

$$5/2 - 3 \times 5/6 = 0$$

$$0 - 0 \times 5/6 = 0$$

$$-1/2 - 2 \times 5/6 = 7/6$$

$$0 - 1 \times 5/6 = -5/6$$

$$1 - 0 \times 5/6 = 1$$

Table 3:

			80	100	0	0	0
Program	Profit/unit	Qty.	X	Y	S ₁	S ₂	S ₃
Y	100	300	0	I	5/6	-1/6	0
X	80	120	I	0	-2/3	1/3	0
S3	0	240	0	0	7/6	-5/6	I
Net evaluation row			0	0	-500/6	+100/6	
					+160/3	-80/3	0
					$= \frac{180}{6}$	$= -\frac{60}{6}$	

All the values of the net evaluation row of Table 3 are either zero or negative, the optimal program has been obtained.

Here X = 120, y = 300 and the maximum profit
 $= 80 \times 120 + 100 \times 300 = 9600 + 30,000$
 $= \text{Rs. } 39,600.$

Question 6

Three grades of coal A, B and C contains phosphorus and ash as impurities. In a particular industrial process, fuel up to 100 ton (maximum) is required which could contain ash not more than 3% and phosphorus not more than .03%. It is desired to maximize the profit while satisfying these conditions. There is an unlimited supply of each grade. The percentage of impurities and the profits of each grade are as follows:

Coal	Phosphorus (%)	Ash (%)	Profit in Rs. (per ton)
A	.02	3.0	12.00
B	.04	2.0	15.00
C	.03	5.0	14.00

You are required to formulate the Linear-programming (LP) model to solve it by using simplex method to determine optimal product mix and profit.

Answer

Let X_1 , X_2 and X_3 respectively be the amounts in tons of grades A, B, and C used. The constraints are

- (i) Phosphorus content must not exceed 0.03%
- $$.02 X_1 + .04 X_2 + .03 X_3 \leq .03 (X_1 + X_2 + X_3)$$
- $$2X_1 + 4 X_2 + 3X_3 \leq 3 (X_1 + X_2 + X_3) \text{ or } -X_1 + X_2 \leq 0$$
- (ii) Ash content must not exceed 3%
- $$3X_1 + 2 X_2 + 5 X_3 \leq 3 (X_1 + X_2 + X_3) \text{ or } -X_2 + 2X_3 \leq 0$$
- (iii) Total quantity of fuel required is not more than 100 tons. $X_1 + X_2 + X_3 \leq 100$

The Mathematical formulation of the problem is

$$\text{Maximize } Z = 12 X_1 + 15X_2 + 14 X_3$$

Subject to the constraints:

$$-X_1 + X_2 \leq 0$$

$$-X_2 + X_3 \leq 0$$

$$X_1 + X_2 + X_3 \leq 100$$

$$X_1, X_2, X_3 > 0$$

Introducing slack variable $X_4 > 0$, $X_5 > 0$, $X_6 > 0$

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			12	15	14	0	0	0
C _b	Y _b	X _b	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆
0	Y ₄	0	-1	1*	0	1	0	0
0	Y ₅	0	0	-1	2	0	1	0
0	Y ₆	100	1	1	1	0	0	1
	Z		-12	-15	-14	0	0	0
C _b	Y _b	X _b	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆
15	Y ₂	0	-1	1	0	1	0	0
0	Y ₅	0	-1	0	2	1	1	0
0	Y ₆	100	2*	0	1	-1	0	1
	Z		-27		-14	15	0	0
C _b	Y _b	X _b	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆
15	Y ₂	50	0	1	1/2	1/2	0	1/2
0	Y ₅	50	0	0	5/2*	1/2	1	1/2
12	Y ₁	50	1	0	1/2	-1/2	0	1/2
	Z		0	0	-1/2	3/2	0	27/2
C _b	Y _b	X _b	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆
15	Y ₂	40	0	1	0	2/5	-1/5	2/5
14	Y ₃	20	0	0	1	1/5	2/5	1/5
12	Y ₁	40	1	0	0	-3/5	-1/5	2/5
	Z		0	0	0	8/5	1/5	68/5

The optimum solution is $X_1 = 40$, $X_2 = 40$ and $X_3 = 20$ with maximum $Z = 1360$.

Question 7

The initial allocation of a transportation problem, alongwith the unit cost of transportation from each origin to destination is given below. You are required to arrive at the minimum transportation cost by the Vogel's Approximation method and check for optimality.

(Hint: Candidates may consider $u_1 = 0$ at Row 1 for initial cell evaluation)

The Transportation Problem

Requirement

		8		6	4	
11	2		8	6	2	18
9	10					
	9		12	9	6	10
			8			
7	6		3	7	7	8
9	2			2		
	3		5	6	11	4
<u>Availability</u>						
12	8		8	8	4	40

Answer

The concept tested in this problem is Degeneracy with respect to the transportation problem. Total of rows and columns = $(4 + 5) = 9$. Hence, the number of allocations = $9 - 1 = 8$. As the actual number of allocation is 7, a 'zero' allocation is called for. To resolve this, an independent cell with least cost should be chosen. R4C2 has the least cost (cost = 3), but this is not independent. The next least cost cell R4C3 (cost = 5) is independent.

	9 C1	2 C2	5 C3	6 C4	2 C5	Total
0R1	11	2	8	6	2	18
0R2	9	9	12	9	6	10
-2R3	7	6	3	7	7	8
0R4	9	3	5	6	11	4
Total	12	8	8	8	4	40

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Forming Equations through allocated cells

Basic equation	Setting R1 = 0 other values
$R1 + C2 = 2$	Setting R1 = 0, C2 = 2
$R1 + C4 = 6$	C4 = 6
$R1 + C5 = 2$	C5 = 2
$R2 + C1 = 9$	R2 = 0
$R3 + C3 = 3$	R3 = -2
$R4 + C1 = 9$	C1 = 9
$R4 + C3 = 5$	C3 = 5
$R4 + C4 = 6$	R4 = 0

Evaluate unallocated cells

$R1C1 = 11 - 0 - 9 = 2$	$R3C1 = 7 + 2 - 9 = 0$
$R1C3 = 8 - 0 - 5 = 3$	$R3C2 = 6 + 2 - 2 = 6$
$R2C2 = 9 - 0 - 2 = 7$	$R3C4 = 7 + 2 - 6 = 7$
$R2C3 = 12 - 0 - 5 = 7$	$R3C5 = 7 + 2 - 2 = 7$
$R2C4 = 9 - 0 - 6 = 3$	$R4C2 = 3 - 0 - 2 = 1$
$R2C5 = 6 - 0 - 2 = 4$	$R4C5 = 11 - 0 - 2 = 9$

Since all the evaluation is 0 or +ve, the optimal solution is obtained.

Optimal cost = $(8 \times 2) + (6 \times 6) + (4 \times 2) + (10 \times 9) + (8 \times 3) + (2 \times 9) + (0 \times 5) + (2 \times 6)$
 $= 16 + 36 + 8 + 90 + 24 + 18 + 10 + 12 = \text{Rs. } 204.$

Note: As regards allocation of the zero values, the solution to the above problem is also obtained by allocating the zero value in other independent cells such as R1C3, R2C2, R2C3, R3C1, R3C2, R3C4, R3C5. In such situation there will be one more iteration.

Question 8

Goods manufactured at 3 plants, A, B and C are required to be transported to sales outlets X, Y and Z. The unit costs of transporting the goods from the plants to the outlets are given below:

The Transportation Problem

<i>Sales outlets \ Plants</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>Total Demand</i>
<i>X</i>	3	9	6	20
<i>Y</i>	4	4	6	40
<i>Z</i>	8	3	5	60
<i>Total supply</i>	40	50	30	120

You are required to:

- (i) Compute the initial allocation by North-West Corner Rule.
- (ii) Compute the initial allocation by Vogel's approximation method and check whether it is optional.
- (iii) State your analysis on the optionality of allocation under North-West corner Rule and Vogel's Approximation method.

Answer

	20	—	—	20
3		9	6	
	20	20	—	40
4		4	6	
	—	30	30	60
8		3	5	
	40	50	30	120

- (i) Initial allocation under NW corner rule is as above.

$$\begin{array}{rcl}
 \text{Initial cost:} & 20 \times 3 & = 60 \\
 & 20 \times 4 & = 80 \\
 & 20 \times 4 & = 80 \\
 & 30 \times 3 & = 90 \\
 & 30 \times 5 & = \underline{150} \\
 & & \underline{460}
 \end{array}$$

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(ii) Initial solution by VAM:

	20	—	—	20	3		
3		9	6				
	20	—		40	0	0	2
4		4	6				
		50	10	60	2	2	2
8		3	5				
	40	50	30				
	1	1	1				
	4	1	1				
		1	1				

Initial solution:	20×3	=	60
	20×4	=	80
	50×3	=	150
	20×6	=	120
	10×5	=	<u>100</u>
			<u>460</u>

Checking for optimality

3			$u_1 = 0$
4		6	$u_2 = 1$

	3	5	$u_3 = 0$
$v_1 = 3$	$v_2 = 3$	$v_3 = 5$	

$u_i + v_j$

	3	5	0
	4		1
3			0
3	3	5	

$$\Delta_{ij} = C_{ij} - (u_i + v_j)$$

	6	1
	0	
5		

$\Delta_{ij} \geq 0 \therefore$ Solution is optimal

Conclusion:

The solution under VAM is optimal with a zero in R_2C_2 which means that the cell C_2R_2 which means that the cell C_2R_2 can come into solution, which will be another optimal solution. Under NWC rule the initial allocation had C_2R_2 and the total cost was the same Rs. 460 as the total cost under optimal VAM solution. Thus, in this problem, both methods have yielded the optimal solution under the 1st allocation. If we do an optimality test for the solution, we will get a zero for Δ_{ij} in C_3R_2 indicating the other optimal solution which was obtained under VAM.

Question 9

State the methods in which initial feasible solution can be arrived at in a transportation problem

Answer

The methods by which initial feasible solution can be arrived at in a transportation model are as under:

- (i) North West Corner Method.
- (ii) Least Cost Method
- (iii) Vogel's Approximation Method (VAM)

Question 10

The cost per unit of transporting goods from the factories X, Y, Z to destinations. A, B and C, and the quantities demanded and supplied are tabulated below. As the company is working out the optimum logistics, the Govt.; has announced a fall in oil prices. The revised unit costs are exactly half the costs given in the table. You are required to evaluate the minimum transportation cost.

<i>Destinations</i>	A	B	C	Supply
<i>Factories</i>				
X	15	9	6	10
Y	21	12	6	10

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Z	6	18	9	10
Demand	10	10	10	30

Answer

The problem may be treated as an assignment problem. The solution will be the same even if prices are halved. Only at the last stage, calculate the minimum cost and divide it by 2 to account for fall in oil prices.

	A	B	C
X	15	9	6
Y	21	12	6
Z	6	18	9

Subtracting Row minimum, we get

	A	B	C
X	9	3	0
Y	15	6	0
Z	0	12	3

Subtracting Column minimum,

	A	B	C
X	9	3	0
Y	15	6	0
Z	0	9	3

No of lines required to cut Zeros = 3

			Cost / u	Units	Cost	Revised Cost
Allocation:	X B	→	9	10	90	45
	Y C	→	6	10	60	30
	Z A	→	6	10	<u>60</u>	<u>30</u>
					<u>210</u>	<u>105</u>

Minimum cost = 105 Rs.

Alternative Solution I

Least Cost Method

	A	B Step V	C		Step I Diff.	Step III
X	15	9 10	6	10	3	3
Y	21	12	5 10	10	6	6
Z	5 10	18	9	10	3	
	10	10	10	30		
Step I Diff.	9	3	10			
Step III Diff.	Step II	3	10			

← Step IV

Initial Feasible solution

X – B

Y – C

Z – A

Test for optimality

No. of allocation = 3

No. of rows $m = 3$, no. of column = 3

$$m + n - 1 = 3 + 3 - 1 = 5$$

2 very small allocation are done to 2 cells of minimum costs, so that , the following table is got:

	A	B	C
X	15	1 9	e 6
Y	21	12	1 6

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Z	1 6	18	e 9
---	-----	----	-----

$$m + n - 1 = 5$$

Now testing for optimality

				u_i
		9	e	0
			6	0
	6		e	0
v_j	6	9	6	

$u_i + v_j$ for unoccupied cells

	A	B	C
X	6	-	-
Y	6	9	-
Z	-	9	-

Diff = $C_{ij} - (u_i + v_j)$

	A	B	C
X	9	-	-
Y	15	3	-
Z	-	9	-

The Transportation Problem

All $\Delta_{ij} > 0$, Hence this is the optimal solution.

	<i>Original Costs</i>	<i>Reduced Costs due to Oil Price</i>	<i>Qty.</i>	<i>Cost</i>
X – B	9	4.5	10	45
Y – C	6	3	10	30
Z – A	6	3	10	<u>30</u>
				<u>105</u>

Total cost of transportation is minimum at Rs.105

Alternative Solution II

	A	B	C		Step I Diff.	Step III Diff.
X	7.5	4.5	8	10	1.5	1.5
Y	11.5	6	3	10	3	3
Z	8	9	1.5	10	1.5	
Qty.	10	10	10	30		
Step I Diff.	4.5	1.5	0			
Step III Diff.	Step II	1.5	0			

Initial Solution :

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	A	B	C
X	7.5	4.5 10	3
Y	11.5	6	3 10
Z	3 10	9	4.5

No. of rows + no. of column – 1

$$m + n - 1 = 5$$

No. of allocation = 3

Hence add 'e' to 2 least cost cells so that

	A	B	C
X	7.5	4.5 10	3 e
Y	11.5	6	3 10
Z	3 10	9	4.5 e

Now $m + n - 1 = 5$

Testing for optimality,

u_i, v_j table

	A	B	C	u_i
X		4.5	e	0
Y			3	
Z	3		e	0
v_j	3	4.5	3	

The Transportation Problem

$u_i + v_j$ for unoccupied cells

	3	-	-
	3	4.5	-
	-	4.5	-

Cij	u _i +v _j						
7.5	-	-		3	-	-	
11.5	6	-		3	4.5	-	
-	9	-		-	4.5	-	

$$\Delta_{ij} = C_{ij} - (u_i + v_j)$$

4.5	-	-	
11.5	1.5	-	
8.5	4.5	-	

All $\Delta_{ij} > 0$. Hence the solution is optimal.

	Qty.	Cost/u	Total Cost
X – B	10	4.5	45
Y – C	10	3	30
Z – A	10	3	30
Total minimum cost at revised oil prices			105

Question 11

How do you know whether an alternative solution exists for a transportation problem?

Answer

The Δ_{ij} matrix = $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Where c_i is the cost matrix and $(u_i + v_j)$ is the cell evaluation matrix for allocated cell.

The Δ_{ij} matrix has one or more 'Zero' elements, indicating that, if that cell is brought into the solution, the optional cost will not change though the allocation changes.

Thus, a 'Zero' element in the Δ_{ij} matrix reveals the possibility of an alternative solution.

Question 12

Explain the term degeneracy in a transportation problem.

Answer

If a basic feasible solution of transportation problem with m origins and n destinations has fewer than $m + n - 1$ positive x_{ij} (occupied cells) the problem is said to be a degenerate transportation problem. Such a situation may be handled by introducing an infinitesimally small allocation e in the least cost and independent cell.

While in the simple computation degeneracy does not cause any serious difficulty, it can cause computational problem in transportation problem. If we apply modified distribution method, then the dual variable u_i and v_j are obtained from the C_{ij} value to locate one or more C_{ij} value which should be equated to corresponding $C_{ij} + V_{ij}$.

EXERCISE

Question 1

A particular product is manufactured in factories A, B, and D: and is sold at centers 1, 2 and 3. The cost in Rs. of product per unit and capacity in kgms per unit time of each plant is given below:

Factory	Coast (Rs.) per unit	Capacity (kgms) per unit
A	12	100
B	15	20
C	11	60
D	13	80

The sale price in Rs. Per unit and the demand is kgms per unit time are as follows:

Sale Centre	Sale price (Rs.) per unit	Demand (Kgms) per unit
1	15	120
2	14	140
3	16	60

Find the optimal sales distribution.

Answer

Total Profit = Rs. 660

Question 2

A Company has four factories F_1 , F_2 , F_3 and F_4 , manufacturing the same product. Production and raw material costs differ from factory to factory and are given in the first two rows of the following table. The Transportation costs from the factories to sales depots S_1 , S_2 and S_3 are given in the next three rows of the table. The production capacity of each factory is given in the last row.

The last two columns in the table given the sales price and the total requirement at each depot:

Item Per unit	Factory				Sales price Per unit	Requirement
	F_1	F_2	F_3	F_4		
Production cost	15	18	14	13	-	-
Raw material cost	10	9	12	9	-	-

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Transportation cost	3	9	5	4	34	80
	1	7	4	5	32	120
	5	8	3	6	31	150
Production capacity	10	150	50	100	-	-

Determine the most profitable production and distribution schedule and the corresponding profit. The surplus should be taken to yield zero profit.

Answer

Profit associated with the optimum Program is Rs. 480.

Question 3

A company has 3 plants and 3 warehouses. The cost of sending a unit from different plants to the warehouses, production at different plants and demand at different warehouses are shown in the following cost matrix table:

Plants	Warehouses A B C	Production
X	8 16 16	152
Y	32 48 32	164
Z	16 32 48	154
Demand	144 204 82	

Determine a transportation schedule, so that the cost is minimized. Assume that the cost in the cost matrix is given in thousand of rupees.

Answer

On calculating Δ_i 's=0, the solution is not unique.

Question 4

Following is the profit matrix based on four factories and three sales depots of the company:

		S_1	S_2	S_3	Availability
Towns	F_1	6	6	1	10
	F_2	-2	-2	-4	150
	F_3	3	2	2	50

The Transportation Problem

	F_4	8	5	3	100
Requirement		80	120	150	

Determine the most profitable distribution schedule and the corresponding profit, assuming no profit in case of surplus production.

Answer

Total Profit = Rs. 480

Question 5

A company produces a small component for all industrial products and distributes it to five wholesalers at a fixed prices of Rs.2.50 per unit. Sales forecasts indicate that monthly deliveries will be 3,000, 3,000, 10,000, 5,000 and 4,000 units to wholesalers 1,2,3,4 and 5 respectively. The monthly production capabilities are 5,000, 10,000, 12,500 at plants 1, 2 and 3 respectively. The direct costs of production of each unit are Rs.1.00 and Rs.0.80 at plants 1, 2 and 3 respectively. The transportation costs of shipping a unit from a plant to a wholesaler are given below:

		1	2	3	4	5
Plant	1	0.05	0.07	0.10	0.15	0.15
	2	0.08	0.06	0.09	0.12	0.14
	3	0.10	0.09	0.08	0.10	0.15

Find how many components each plant supplies to each wholesaler in order to maximize profit.

Answer

Profit = Rs.32,520

Question 6

The following table shows all the necessary information on the available supply to each warehouse, the requirement of each market and the unit transportation cost from each warehouse to each market:

		Market				Supply
		I	II	III	IV	
Warehouse	A	5	2	4	3	22
	B	4	8	1	6	15

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	C	4	6	7	5	8
Requirement		7	12	17	9	

The shipping clerk has worked out the following schedule from his experience:

12 Units from A to II

1 Unit from A to III

9 Units from A to IV

15 Units from B to III

7 Units from C to I and

1 Unit from C to III

You are required to answer the following:

- Check and see if the clerk has the optimal schedule;
- Find the optimal schedule and minimum total shipping cost; and
- If the clerk is approached by a carrier of route C to II, who offers to reduce his rate in the hope of getting some business, by how much should the rate be reduced before the clerk should consider giving him an order?

Answer

Total Shipping Cost = Rs.103.

Question 7

A company has three warehouses W_1 , W_2 and W_3 . It is required to deliver a product from these warehouses to three customers A, B and C. These warehouses have the following units in stock.

Warehouse:	W_1	W_2	W_3
No. of units:	65	42	43

and customer requirements are:

Customer:	A	B	C
No. of units:	70	30	50

The Transportation Problem

The table below shows the costs of transporting one unit from warehouse to the customer:

		Warehouse		
		W_1	W_2	W_3
Customer	A	5	7	8
	B	4	4	6
	C	6	7	7

Find the optimal transportation route.

Answer Total Cost = Rs. 830

Question 8

A company has four factories situated in four different locations in the country and four sales agencies located in four other locations in the country. The cost of production (Rs. Per unit), the sales price (Rs. per unit), and shipping cost (Rs. Per unit) in the case of matrix, monthly capacities and monthly requirements are given below:

Factory	Sales Agency				Monthly Capacity (Units)	Cost of production
	1	2	3	4		
A	7	5	6	4	10	10
B	3	5	4	2	15	15
C	4	6	4	5	20	16
D	8	7	6	5	15	15
Monthly Requirement (Units)	8	12	18	22		
Sales Price	20	22	25	18		

Find the monthly production and distribution schedule which will maximize profit.

Answer

Since one of the Δ_{ij} 's is Zero, the optimal solution obtained above is not unique. Alternate solution also exists.

Question 9

XYZ and Co. has provided the following data seeking your advice on optimum investment strategy.

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Investment made at the Beginning of year	Net Return Data (in Paise) of Selected Investment				Amount available (Lacs)
	P	Q	R	S	
1	95	80	70	60	70
2	75	65	60	50	40
3	70	45	50	40	90
4	60	40	40	30	30
Maximum Investment (Lacs)	40	30	60	60	

The following additional information are also provided

- P, Q, R and S represent the selected investments,
- The company has decided to have four years investment plan.
- The policy of the company is that amount invested in any year will remain so until the end of the fourth year.
- The values (Paise) in the table represent net return on investment of one Rupee till the end of the planning horizon (for example, a Rupee investment in Investment P at the beginning of year 1 will grow to Rs.1.95 by the end of the fourth year, yielding a return of 95 paise)

Using the above determine the optimum investment strategy.

Answer

The optimal allocations are given below:

Year	Invest in	Net Return
1	Invest Rs 40 lacs in investment P	$0.95 \times \text{Rs.} 40 \text{ lacs} = \text{Rs. } 38,00,000$
	Rs 30 lacs in investment Q	$0.80 \times \text{Rs.} 30 \text{ lacs} = \text{Rs. } 24,00,000$
2	Invest Rs 20 lacs in investment Q	$0.65 \times \text{Rs.} 20 \text{ lacs} = \text{Rs. } 13,00,000$
	Rs 20 lacs in investment R	$0.60 \times \text{Rs.} 20 \text{ lacs} = \text{Rs. } 12,00,000$
3	Invest Rs 40 lacs in investment R	$0.50 \times \text{Rs.} 40 \text{ lacs} = \text{Rs. } 20,00,000$
	Rs 50 lacs in investment S	$0.40 \times \text{Rs.} 50 \text{ lacs} = \text{Rs. } 20,00,000$
4	Invest Rs.10 lacs in investment S	$0.30 \times \text{Rs.} 10 \text{ lacs} = \text{Rs. } 3,00,000$
	Total	Rs.130,00,000

The Transportation Problem

Question 10

A company has four terminals U, V, W and X. At the start of a particular day 10, 4, 6 and 5 trailers respectively are available at these terminals. During the previous night 13, 10, 6 and 6 trailers respectively were loaded at plants A, B, C and D. The company dispatcher has come up with the costs between the terminals and plants as follows:

		Plants			
		A	B	C	D
Terminals	U	20	36	10	28
	V	40	20	45	20
	W	75	35	45	50
	X	30	35	40	25

Find the allocation of loaded trailers from plants to terminals in order to minimize transportation cost.

Answer Terminal Plant Cost = Rs. 555